# Expected Number of Ordered Sub-arrays

## Definitions

Ordered Sub-array :

Given an array ***A*** of length ***n***, the range ***[i, j)*** is said to be an Ordered Sub-array of ***A***, if

1. ***A[k+1] > A[k]***, for ***i <= k < j***;

2. ***A[i – 1] > A[i]***, if ***i > 0***;

3. ***A[j – 1] > A[j]***, if ***j < n***;

## Problems

An array ***A*** is said to be random if the element in ***A*** is random and the order of the element is random. Assume array ***A*** is random and the elements in it are distinct, what is the expected number of Ordered Sub-arrays in ***A***?

## Solutions

A permutation of an array is an ordering of the elements in the array.

For a random array of length ***n*** with distinct elements, all the ***n!*** permutations of the array are equally likely to occur.

Let denote the number of permutations of an array of length ***n*** , where each of the permutations has ***i*** Ordered Sub-arrays.

Let ***A*** be an array of length ***(n+1)***.

Let ***A'*** be the array formed from ***A*** by removing the smallest element in ***A***.

Let denote a permutation of ***A***, where there are ***i*** Ordered Sub-arrays.

Let denote a permutation of ***A'***, where there are ***i*** Ordered Sub-arrays.

For every . If the smallest element is inserted at the start of an Ordered Sub-array in , we will get , there are ***i*** such cases. If the smallest element is inserted at the other positions instead of the start of an Ordered Sub-array, we will get , there are ***(n + 2 – i)*** such cases.

For every . If the smallest element is removed from , we will either get or .

Since is the number of , is the number of , andis the number of , follow from the above reasoning:

***Equation 1***

Let denote the possibility that there are ***i*** Ordered Sub-arrays in a random array, then , by “***Equation 1***” :

***Equation 2***

Let ***E(n)*** denote the expected number of Ordered Sub-arrays in a random array, then

, by “***Equation 2***”:

That is the expected number of Ordered Sub-arrays in an array of length ***n*** is ***(n+1)/2***.

## Codes

/\*

\* OrderedSubArrayTest.cpp

\*

\* Note :

\* C++11 feature is used. a compiler that support C++11 features is needed.

\*

\* Created on: Jan 4, 2014

\* Author: Stiff Liu

\*/

**#include** <iostream>

**#include** <cassert>

**#include** <algorithm>

**using** **namespace** std;

**struct** OrderedSubArrayTest {

/\*\*

\* Find the first index, say {@var i}, in an array of length n, starting from index {@var offset},

\* such that the range [offset, i) is in increasing order, but the range [offset, i] is not in increasing order.

\* If the range [offset, n) is in increasing order(we define that an empty range is always in increasing order),

\* then offset will be returned.

\* We define this index as : first disorder index.

\*

\* The cost of this algorithm is linear.

\*

\* @param values The array bo be searched.

\* @param n Number of elements in the array.

\* @param comparator The "less than" relation between two elements in the array

\* @param offset The index from which to start the search.

\* @return The first disorder index.

\*/

**template**<**class** **T**, **class** **Comparator**>

**static** **unsigned** **int** **firstDisorderIndex**(**T** values, **unsigned** **int** n, **Comparator** comparator,

**unsigned** **int** offset = 0) {

**for** (**unsigned** **int** i = offset + 1; i < n; ++i)

**if** (comparator(values[i], values[i - 1]))

**return** i;

**return** offset;

}

/\*\*

\* Find the number of ordered sub-arrays.

\*

\* An ordered sub-array of an array {@var A}, starting from index {@var i} and ending at index {@var j},

\* is the range [i, j) and A[i] < A[i - 1] if i > 0, A[j] < A[j - 1].

\*

\* The cost of this algorithm is linear.

\*

\* @param values The array to be searched.

\* @param n Number of elements in the array.

\* @param comparator The "less than" relation between two elements.

\*/

**template**<**class** **T**, **class** **Comparator**>

**static** **unsigned** **int** **numberOfOrderedSubarrays**(**T** values, **unsigned** **int** n,

**Comparator** comparator) {

**if** (n == 0)

**return** 0;

**unsigned** **int** count = 0;

**unsigned** **int** i = 0;

**unsigned** **int** tmp = 0;

**while** ((tmp = firstDisorderIndex(values, n, comparator, i)) != i) {

++count;

i = tmp;

}

++count;

**return** count;

}

/\*\*

\* Count the number of cases, denoted by C(n, i), that there are i ordered subarrays in an array of length n.

\*

\* @param n Length of an array.

\* @param counts {@code counts[i]} denote the number of cases that that there

\* are (i+1) ordered subarray in an array of length {@code n}

\*

\* Note :

\* The following recurrence relation is used :

\* C(n + 1, i) = i \* C(n, i) + (n + 2 - i) \* C(n, i - 1), for 2 <= i <= n

\* C(n + 1, 1) = C(n + 1, n + 1) = 1

\*/

**static** **void** **count**(**unsigned** **int** n, **unsigned** **long** **long** \*counts) {

**if** (n == 0)

**return**;

counts[0] = 1;

**if** (n == 1)

**return**;

counts[1] = 1;

**for** (**unsigned** **int** j = 3; j <= n; ++j) {

**unsigned** **long** **long** last = counts[0];

**for** (**unsigned** **int** i = 1; i < j - 1; ++i) {

**unsigned** **long** **long** tmp = counts[i];

counts[i] = i \* tmp + (j + 1 - i) \* last;

last = tmp;

}

counts[j - 1] = 1;

}

}

**static** **void** **validateAlgorithm**() {

**unsigned** **int** values1[] = { 0, 1, 2, 3, 4 };

**unsigned** **int** values2[] = { 4, 3, 2, 1, 0 };

**unsigned** **int** values3[] = { 0, 3, 1, 2, 5, 4 };

**unsigned** **int** values4[] = { 1, 1, 1, 0, 0, 0, 1, 1, 0 };

assert(

*numberOfOrderedSubarrays*(values1,

(**sizeof** values1 / **sizeof**(\*values1)),

std::less<**unsigned** **int**>()) == 1);

assert(

*numberOfOrderedSubarrays*(values2,

(**sizeof** values2 / **sizeof**(\*values2)),

std::less<**unsigned** **int**>()) == 5);

assert(

*numberOfOrderedSubarrays*(values3,

(**sizeof** values3 / **sizeof**(\*values3)),

std::less<**unsigned** **int**>()) == 3);

assert(

*numberOfOrderedSubarrays*(values4,

(**sizeof** values4 / **sizeof**(\*values4)),

std::less<**unsigned** **int**>()) == 3);

}

/\*\*

\* Given an randomly generated array of length n, what's the expected number of ordered subarrays in it?

\* Denote the expected number of ordered subarrays in an array of length n as E(n).

\* Then we will have the following recurrence relation:

\* E(n+1) = (n / (n+1))E(n) + 1;

\* The base case is E(1) = 1.

\* After solving this recurrence relation, we will have

\* E(n) = (n + 1) / 2

\*

\*/

**static** **int** **test**(**unsigned** **int** argc, **char** \*argv[]) {

*validateAlgorithm*();

**unsigned** **int** start = 3;

**while** (start <= 10000) {

**unsigned** **int** values[start];

**unsigned** **int** iteration = 1;

**unsigned** **int** count = 0;

**for** (**unsigned** **int** i = 0; i < start; ++i)

values[i] = i;

**for** (**unsigned** **int** i = 0; i < iteration; ++i) {

std::random\_shuffle(values, values + start);

count += *numberOfOrderedSubarrays*(values, start,

std::less<**unsigned** **int**>());

}

**double** average = count / (**double**) iteration;

cout << "number of elements : " << start << ", expected : " << (start + 1) / 2 << ", actual : " << average

<< ", ratio : " << average \* 2 / (start + 1) << **endl**;

start += 10;

}

**return** 0;

}

};

**int** **main**(**int** argc, **char** \*argv[]) {

**return** OrderedSubArrayTest::*test*(argc, argv);

}

## Sample Output

number of elements : 3, expected : 2, actual : 2, ratio : 1

number of elements : 13, expected : 7, actual : 7, ratio : 1

number of elements : 23, expected : 12, actual : 11, ratio : 0.916667

number of elements : 33, expected : 17, actual : 15, ratio : 0.882353

number of elements : 43, expected : 22, actual : 22, ratio : 1

number of elements : 53, expected : 27, actual : 24, ratio : 0.888889

number of elements : 63, expected : 32, actual : 33, ratio : 1.03125

number of elements : 73, expected : 37, actual : 35, ratio : 0.945946

number of elements : 83, expected : 42, actual : 39, ratio : 0.928571

number of elements : 93, expected : 47, actual : 43, ratio : 0.914894

number of elements : 103, expected : 52, actual : 49, ratio : 0.942308

number of elements : 113, expected : 57, actual : 58, ratio : 1.01754

number of elements : 123, expected : 62, actual : 60, ratio : 0.967742

number of elements : 133, expected : 67, actual : 70, ratio : 1.04478

number of elements : 143, expected : 72, actual : 71, ratio : 0.986111

number of elements : 153, expected : 77, actual : 75, ratio : 0.974026

number of elements : 163, expected : 82, actual : 85, ratio : 1.03659

number of elements : 173, expected : 87, actual : 88, ratio : 1.01149

number of elements : 183, expected : 92, actual : 91, ratio : 0.98913

number of elements : 193, expected : 97, actual : 95, ratio : 0.979381

number of elements : 203, expected : 102, actual : 104, ratio : 1.01961

number of elements : 213, expected : 107, actual : 113, ratio : 1.05607

number of elements : 223, expected : 112, actual : 112, ratio : 1

number of elements : 233, expected : 117, actual : 114, ratio : 0.974359

number of elements : 243, expected : 122, actual : 124, ratio : 1.01639

number of elements : 253, expected : 127, actual : 134, ratio : 1.05512

number of elements : 263, expected : 132, actual : 132, ratio : 1

number of elements : 273, expected : 137, actual : 137, ratio : 1

number of elements : 283, expected : 142, actual : 147, ratio : 1.03521

number of elements : 293, expected : 147, actual : 135, ratio : 0.918367

number of elements : 303, expected : 152, actual : 163, ratio : 1.07237

number of elements : 313, expected : 157, actual : 149, ratio : 0.949045

number of elements : 323, expected : 162, actual : 156, ratio : 0.962963

number of elements : 333, expected : 167, actual : 161, ratio : 0.964072

number of elements : 343, expected : 172, actual : 175, ratio : 1.01744

number of elements : 353, expected : 177, actual : 177, ratio : 1

number of elements : 363, expected : 182, actual : 175, ratio : 0.961538

number of elements : 373, expected : 187, actual : 191, ratio : 1.02139

number of elements : 383, expected : 192, actual : 193, ratio : 1.00521

number of elements : 393, expected : 197, actual : 195, ratio : 0.989848

number of elements : 403, expected : 202, actual : 201, ratio : 0.99505

number of elements : 413, expected : 207, actual : 217, ratio : 1.04831

number of elements : 423, expected : 212, actual : 207, ratio : 0.976415

number of elements : 433, expected : 217, actual : 209, ratio : 0.963134

number of elements : 443, expected : 222, actual : 218, ratio : 0.981982

number of elements : 453, expected : 227, actual : 225, ratio : 0.991189

number of elements : 463, expected : 232, actual : 228, ratio : 0.982759

number of elements : 473, expected : 237, actual : 240, ratio : 1.01266

number of elements : 483, expected : 242, actual : 251, ratio : 1.03719

number of elements : 493, expected : 247, actual : 254, ratio : 1.02834

number of elements : 503, expected : 252, actual : 242, ratio : 0.960317

number of elements : 513, expected : 257, actual : 255, ratio : 0.992218

number of elements : 523, expected : 262, actual : 261, ratio : 0.996183

number of elements : 533, expected : 267, actual : 265, ratio : 0.992509

number of elements : 543, expected : 272, actual : 271, ratio : 0.996324

number of elements : 553, expected : 277, actual : 274, ratio : 0.98917

number of elements : 563, expected : 282, actual : 290, ratio : 1.02837

number of elements : 573, expected : 287, actual : 292, ratio : 1.01742

number of elements : 583, expected : 292, actual : 299, ratio : 1.02397

number of elements : 593, expected : 297, actual : 297, ratio : 1

number of elements : 603, expected : 302, actual : 297, ratio : 0.983444

number of elements : 613, expected : 307, actual : 305, ratio : 0.993485

number of elements : 623, expected : 312, actual : 322, ratio : 1.03205

number of elements : 633, expected : 317, actual : 311, ratio : 0.981073

number of elements : 643, expected : 322, actual : 311, ratio : 0.965839

number of elements : 653, expected : 327, actual : 340, ratio : 1.03976

number of elements : 663, expected : 332, actual : 332, ratio : 1

number of elements : 673, expected : 337, actual : 336, ratio : 0.997033

number of elements : 683, expected : 342, actual : 332, ratio : 0.97076

number of elements : 693, expected : 347, actual : 347, ratio : 1

number of elements : 703, expected : 352, actual : 353, ratio : 1.00284

number of elements : 713, expected : 357, actual : 376, ratio : 1.05322

number of elements : 723, expected : 362, actual : 357, ratio : 0.986188

number of elements : 733, expected : 367, actual : 369, ratio : 1.00545

number of elements : 743, expected : 372, actual : 372, ratio : 1

number of elements : 753, expected : 377, actual : 384, ratio : 1.01857

number of elements : 763, expected : 382, actual : 385, ratio : 1.00785

number of elements : 773, expected : 387, actual : 377, ratio : 0.97416

number of elements : 783, expected : 392, actual : 409, ratio : 1.04337

number of elements : 793, expected : 397, actual : 379, ratio : 0.95466

number of elements : 803, expected : 402, actual : 404, ratio : 1.00498

number of elements : 813, expected : 407, actual : 388, ratio : 0.953317

number of elements : 823, expected : 412, actual : 408, ratio : 0.990291

number of elements : 833, expected : 417, actual : 421, ratio : 1.00959

number of elements : 843, expected : 422, actual : 418, ratio : 0.990521

number of elements : 853, expected : 427, actual : 416, ratio : 0.974239

number of elements : 863, expected : 432, actual : 434, ratio : 1.00463

number of elements : 873, expected : 437, actual : 442, ratio : 1.01144

number of elements : 883, expected : 442, actual : 444, ratio : 1.00452

number of elements : 893, expected : 447, actual : 461, ratio : 1.03132

number of elements : 903, expected : 452, actual : 449, ratio : 0.993363

number of elements : 913, expected : 457, actual : 464, ratio : 1.01532

number of elements : 923, expected : 462, actual : 451, ratio : 0.97619

number of elements : 933, expected : 467, actual : 478, ratio : 1.02355

number of elements : 943, expected : 472, actual : 466, ratio : 0.987288

number of elements : 953, expected : 477, actual : 467, ratio : 0.979036

number of elements : 963, expected : 482, actual : 490, ratio : 1.0166

number of elements : 973, expected : 487, actual : 486, ratio : 0.997947

number of elements : 983, expected : 492, actual : 485, ratio : 0.985772

number of elements : 993, expected : 497, actual : 506, ratio : 1.01811

number of elements : 1003, expected : 502, actual : 498, ratio : 0.992032

number of elements : 1013, expected : 507, actual : 500, ratio : 0.986193

number of elements : 1023, expected : 512, actual : 512, ratio : 1

number of elements : 1033, expected : 517, actual : 511, ratio : 0.988395

number of elements : 1043, expected : 522, actual : 515, ratio : 0.98659

number of elements : 1053, expected : 527, actual : 528, ratio : 1.0019

number of elements : 1063, expected : 532, actual : 533, ratio : 1.00188

number of elements : 1073, expected : 537, actual : 527, ratio : 0.981378

number of elements : 1083, expected : 542, actual : 537, ratio : 0.990775

number of elements : 1093, expected : 547, actual : 537, ratio : 0.981718

number of elements : 1103, expected : 552, actual : 557, ratio : 1.00906

number of elements : 1113, expected : 557, actual : 570, ratio : 1.02334

number of elements : 1123, expected : 562, actual : 555, ratio : 0.987544

number of elements : 1133, expected : 567, actual : 557, ratio : 0.982363

number of elements : 1143, expected : 572, actual : 571, ratio : 0.998252

number of elements : 1153, expected : 577, actual : 572, ratio : 0.991334

number of elements : 1163, expected : 582, actual : 578, ratio : 0.993127

number of elements : 1173, expected : 587, actual : 607, ratio : 1.03407

number of elements : 1183, expected : 592, actual : 598, ratio : 1.01014

number of elements : 1193, expected : 597, actual : 601, ratio : 1.0067

number of elements : 1203, expected : 602, actual : 598, ratio : 0.993355

number of elements : 1213, expected : 607, actual : 605, ratio : 0.996705

number of elements : 1223, expected : 612, actual : 629, ratio : 1.02778

number of elements : 1233, expected : 617, actual : 595, ratio : 0.964344

number of elements : 1243, expected : 622, actual : 628, ratio : 1.00965

number of elements : 1253, expected : 627, actual : 637, ratio : 1.01595

number of elements : 1263, expected : 632, actual : 625, ratio : 0.988924

number of elements : 1273, expected : 637, actual : 639, ratio : 1.00314

number of elements : 1283, expected : 642, actual : 649, ratio : 1.0109

number of elements : 1293, expected : 647, actual : 647, ratio : 1

number of elements : 1303, expected : 652, actual : 656, ratio : 1.00613

number of elements : 1313, expected : 657, actual : 656, ratio : 0.998478

number of elements : 1323, expected : 662, actual : 646, ratio : 0.975831

number of elements : 1333, expected : 667, actual : 655, ratio : 0.982009

number of elements : 1343, expected : 672, actual : 658, ratio : 0.979167

number of elements : 1353, expected : 677, actual : 690, ratio : 1.0192

number of elements : 1363, expected : 682, actual : 672, ratio : 0.985337

number of elements : 1373, expected : 687, actual : 686, ratio : 0.998544

number of elements : 1383, expected : 692, actual : 698, ratio : 1.00867

number of elements : 1393, expected : 697, actual : 679, ratio : 0.974175

number of elements : 1403, expected : 702, actual : 714, ratio : 1.01709

number of elements : 1413, expected : 707, actual : 699, ratio : 0.988685

number of elements : 1423, expected : 712, actual : 715, ratio : 1.00421

number of elements : 1433, expected : 717, actual : 713, ratio : 0.994421

number of elements : 1443, expected : 722, actual : 732, ratio : 1.01385

number of elements : 1453, expected : 727, actual : 731, ratio : 1.0055

number of elements : 1463, expected : 732, actual : 736, ratio : 1.00546

number of elements : 1473, expected : 737, actual : 736, ratio : 0.998643

number of elements : 1483, expected : 742, actual : 738, ratio : 0.994609

number of elements : 1493, expected : 747, actual : 758, ratio : 1.01473

number of elements : 1503, expected : 752, actual : 749, ratio : 0.996011

number of elements : 1513, expected : 757, actual : 767, ratio : 1.01321

number of elements : 1523, expected : 762, actual : 764, ratio : 1.00262

number of elements : 1533, expected : 767, actual : 770, ratio : 1.00391

number of elements : 1543, expected : 772, actual : 777, ratio : 1.00648

number of elements : 1553, expected : 777, actual : 760, ratio : 0.978121

number of elements : 1563, expected : 782, actual : 760, ratio : 0.971867

number of elements : 1573, expected : 787, actual : 785, ratio : 0.997459

number of elements : 1583, expected : 792, actual : 772, ratio : 0.974747

number of elements : 1593, expected : 797, actual : 782, ratio : 0.981179

number of elements : 1603, expected : 802, actual : 827, ratio : 1.03117

number of elements : 1613, expected : 807, actual : 801, ratio : 0.992565

number of elements : 1623, expected : 812, actual : 812, ratio : 1

number of elements : 1633, expected : 817, actual : 825, ratio : 1.00979

number of elements : 1643, expected : 822, actual : 830, ratio : 1.00973

number of elements : 1653, expected : 827, actual : 826, ratio : 0.998791

number of elements : 1663, expected : 832, actual : 828, ratio : 0.995192

number of elements : 1673, expected : 837, actual : 829, ratio : 0.990442

number of elements : 1683, expected : 842, actual : 831, ratio : 0.986936

number of elements : 1693, expected : 847, actual : 824, ratio : 0.972845

number of elements : 1703, expected : 852, actual : 853, ratio : 1.00117

number of elements : 1713, expected : 857, actual : 845, ratio : 0.985998

number of elements : 1723, expected : 862, actual : 872, ratio : 1.0116

number of elements : 1733, expected : 867, actual : 879, ratio : 1.01384

number of elements : 1743, expected : 872, actual : 858, ratio : 0.983945

number of elements : 1753, expected : 877, actual : 895, ratio : 1.02052

number of elements : 1763, expected : 882, actual : 879, ratio : 0.996599

number of elements : 1773, expected : 887, actual : 888, ratio : 1.00113

number of elements : 1783, expected : 892, actual : 891, ratio : 0.998879

number of elements : 1793, expected : 897, actual : 877, ratio : 0.977703

number of elements : 1803, expected : 902, actual : 886, ratio : 0.982262

number of elements : 1813, expected : 907, actual : 945, ratio : 1.0419

number of elements : 1823, expected : 912, actual : 913, ratio : 1.0011

number of elements : 1833, expected : 917, actual : 910, ratio : 0.992366

number of elements : 1843, expected : 922, actual : 921, ratio : 0.998915

number of elements : 1853, expected : 927, actual : 914, ratio : 0.985976

number of elements : 1863, expected : 932, actual : 939, ratio : 1.00751

number of elements : 1873, expected : 937, actual : 934, ratio : 0.996798

number of elements : 1883, expected : 942, actual : 934, ratio : 0.991507

number of elements : 1893, expected : 947, actual : 951, ratio : 1.00422

number of elements : 1903, expected : 952, actual : 952, ratio : 1

number of elements : 1913, expected : 957, actual : 962, ratio : 1.00522

number of elements : 1923, expected : 962, actual : 977, ratio : 1.01559

number of elements : 1933, expected : 967, actual : 974, ratio : 1.00724

number of elements : 1943, expected : 972, actual : 950, ratio : 0.977366

number of elements : 1953, expected : 977, actual : 960, ratio : 0.9826

number of elements : 1963, expected : 982, actual : 987, ratio : 1.00509

number of elements : 1973, expected : 987, actual : 967, ratio : 0.979737

number of elements : 1983, expected : 992, actual : 1012, ratio : 1.02016

number of elements : 1993, expected : 997, actual : 1029, ratio : 1.0321

number of elements : 2003, expected : 1002, actual : 986, ratio : 0.984032

number of elements : 2013, expected : 1007, actual : 1037, ratio : 1.02979

number of elements : 2023, expected : 1012, actual : 991, ratio : 0.979249

number of elements : 2033, expected : 1017, actual : 1039, ratio : 1.02163

number of elements : 2043, expected : 1022, actual : 1010, ratio : 0.988258

number of elements : 2053, expected : 1027, actual : 1030, ratio : 1.00292

number of elements : 2063, expected : 1032, actual : 1031, ratio : 0.999031

number of elements : 2073, expected : 1037, actual : 1019, ratio : 0.982642

number of elements : 2083, expected : 1042, actual : 1044, ratio : 1.00192

number of elements : 2093, expected : 1047, actual : 1030, ratio : 0.983763

number of elements : 2103, expected : 1052, actual : 1050, ratio : 0.998099

number of elements : 2113, expected : 1057, actual : 1055, ratio : 0.998108

number of elements : 2123, expected : 1062, actual : 1067, ratio : 1.00471

number of elements : 2133, expected : 1067, actual : 1056, ratio : 0.989691

number of elements : 2143, expected : 1072, actual : 1061, ratio : 0.989739

number of elements : 2153, expected : 1077, actual : 1086, ratio : 1.00836

number of elements : 2163, expected : 1082, actual : 1064, ratio : 0.983364

number of elements : 2173, expected : 1087, actual : 1117, ratio : 1.0276

number of elements : 2183, expected : 1092, actual : 1089, ratio : 0.997253

number of elements : 2193, expected : 1097, actual : 1110, ratio : 1.01185

number of elements : 2203, expected : 1102, actual : 1098, ratio : 0.99637

number of elements : 2213, expected : 1107, actual : 1138, ratio : 1.028

number of elements : 2223, expected : 1112, actual : 1123, ratio : 1.00989

number of elements : 2233, expected : 1117, actual : 1122, ratio : 1.00448

number of elements : 2243, expected : 1122, actual : 1122, ratio : 1

number of elements : 2253, expected : 1127, actual : 1145, ratio : 1.01597

number of elements : 2263, expected : 1132, actual : 1127, ratio : 0.995583

number of elements : 2273, expected : 1137, actual : 1134, ratio : 0.997361

number of elements : 2283, expected : 1142, actual : 1153, ratio : 1.00963

number of elements : 2293, expected : 1147, actual : 1145, ratio : 0.998256

number of elements : 2303, expected : 1152, actual : 1129, ratio : 0.980035

number of elements : 2313, expected : 1157, actual : 1153, ratio : 0.996543

number of elements : 2323, expected : 1162, actual : 1183, ratio : 1.01807

number of elements : 2333, expected : 1167, actual : 1194, ratio : 1.02314

number of elements : 2343, expected : 1172, actual : 1156, ratio : 0.986348

number of elements : 2353, expected : 1177, actual : 1180, ratio : 1.00255

number of elements : 2363, expected : 1182, actual : 1203, ratio : 1.01777

number of elements : 2373, expected : 1187, actual : 1205, ratio : 1.01516

number of elements : 2383, expected : 1192, actual : 1184, ratio : 0.993289

number of elements : 2393, expected : 1197, actual : 1228, ratio : 1.0259

number of elements : 2403, expected : 1202, actual : 1212, ratio : 1.00832

number of elements : 2413, expected : 1207, actual : 1223, ratio : 1.01326

number of elements : 2423, expected : 1212, actual : 1204, ratio : 0.993399

number of elements : 2433, expected : 1217, actual : 1216, ratio : 0.999178

number of elements : 2443, expected : 1222, actual : 1209, ratio : 0.989362

number of elements : 2453, expected : 1227, actual : 1226, ratio : 0.999185

number of elements : 2463, expected : 1232, actual : 1214, ratio : 0.98539

number of elements : 2473, expected : 1237, actual : 1220, ratio : 0.986257

number of elements : 2483, expected : 1242, actual : 1248, ratio : 1.00483

number of elements : 2493, expected : 1247, actual : 1232, ratio : 0.987971

number of elements : 2503, expected : 1252, actual : 1279, ratio : 1.02157

number of elements : 2513, expected : 1257, actual : 1277, ratio : 1.01591

number of elements : 2523, expected : 1262, actual : 1283, ratio : 1.01664

number of elements : 2533, expected : 1267, actual : 1253, ratio : 0.98895

number of elements : 2543, expected : 1272, actual : 1296, ratio : 1.01887

number of elements : 2553, expected : 1277, actual : 1286, ratio : 1.00705

number of elements : 2563, expected : 1282, actual : 1259, ratio : 0.982059

number of elements : 2573, expected : 1287, actual : 1274, ratio : 0.989899

number of elements : 2583, expected : 1292, actual : 1253, ratio : 0.969814

number of elements : 2593, expected : 1297, actual : 1295, ratio : 0.998458

number of elements : 2603, expected : 1302, actual : 1283, ratio : 0.985407

number of elements : 2613, expected : 1307, actual : 1314, ratio : 1.00536

number of elements : 2623, expected : 1312, actual : 1317, ratio : 1.00381

number of elements : 2633, expected : 1317, actual : 1299, ratio : 0.986333

number of elements : 2643, expected : 1322, actual : 1340, ratio : 1.01362

number of elements : 2653, expected : 1327, actual : 1319, ratio : 0.993971

number of elements : 2663, expected : 1332, actual : 1299, ratio : 0.975225

number of elements : 2673, expected : 1337, actual : 1337, ratio : 1

number of elements : 2683, expected : 1342, actual : 1318, ratio : 0.982116

number of elements : 2693, expected : 1347, actual : 1351, ratio : 1.00297

number of elements : 2703, expected : 1352, actual : 1363, ratio : 1.00814

number of elements : 2713, expected : 1357, actual : 1357, ratio : 1

number of elements : 2723, expected : 1362, actual : 1381, ratio : 1.01395

number of elements : 2733, expected : 1367, actual : 1366, ratio : 0.999268

number of elements : 2743, expected : 1372, actual : 1381, ratio : 1.00656

number of elements : 2753, expected : 1377, actual : 1375, ratio : 0.998548

number of elements : 2763, expected : 1382, actual : 1389, ratio : 1.00507

number of elements : 2773, expected : 1387, actual : 1419, ratio : 1.02307

number of elements : 2783, expected : 1392, actual : 1409, ratio : 1.01221

number of elements : 2793, expected : 1397, actual : 1389, ratio : 0.994273

number of elements : 2803, expected : 1402, actual : 1404, ratio : 1.00143

number of elements : 2813, expected : 1407, actual : 1403, ratio : 0.997157

number of elements : 2823, expected : 1412, actual : 1413, ratio : 1.00071

number of elements : 2833, expected : 1417, actual : 1419, ratio : 1.00141

number of elements : 2843, expected : 1422, actual : 1440, ratio : 1.01266

number of elements : 2853, expected : 1427, actual : 1441, ratio : 1.00981

number of elements : 2863, expected : 1432, actual : 1443, ratio : 1.00768

number of elements : 2873, expected : 1437, actual : 1425, ratio : 0.991649

number of elements : 2883, expected : 1442, actual : 1433, ratio : 0.993759

number of elements : 2893, expected : 1447, actual : 1407, ratio : 0.972357

number of elements : 2903, expected : 1452, actual : 1449, ratio : 0.997934

number of elements : 2913, expected : 1457, actual : 1449, ratio : 0.994509

number of elements : 2923, expected : 1462, actual : 1450, ratio : 0.991792

number of elements : 2933, expected : 1467, actual : 1493, ratio : 1.01772

number of elements : 2943, expected : 1472, actual : 1457, ratio : 0.98981

number of elements : 2953, expected : 1477, actual : 1478, ratio : 1.00068

number of elements : 2963, expected : 1482, actual : 1489, ratio : 1.00472

number of elements : 2973, expected : 1487, actual : 1503, ratio : 1.01076

number of elements : 2983, expected : 1492, actual : 1503, ratio : 1.00737

number of elements : 2993, expected : 1497, actual : 1493, ratio : 0.997328

number of elements : 3003, expected : 1502, actual : 1502, ratio : 1

number of elements : 3013, expected : 1507, actual : 1511, ratio : 1.00265

number of elements : 3023, expected : 1512, actual : 1504, ratio : 0.994709

number of elements : 3033, expected : 1517, actual : 1524, ratio : 1.00461

number of elements : 3043, expected : 1522, actual : 1525, ratio : 1.00197

number of elements : 3053, expected : 1527, actual : 1550, ratio : 1.01506

number of elements : 3063, expected : 1532, actual : 1555, ratio : 1.01501

number of elements : 3073, expected : 1537, actual : 1553, ratio : 1.01041

number of elements : 3083, expected : 1542, actual : 1554, ratio : 1.00778

number of elements : 3093, expected : 1547, actual : 1558, ratio : 1.00711

number of elements : 3103, expected : 1552, actual : 1557, ratio : 1.00322

number of elements : 3113, expected : 1557, actual : 1596, ratio : 1.02505

number of elements : 3123, expected : 1562, actual : 1584, ratio : 1.01408

number of elements : 3133, expected : 1567, actual : 1582, ratio : 1.00957

number of elements : 3143, expected : 1572, actual : 1568, ratio : 0.997455

number of elements : 3153, expected : 1577, actual : 1586, ratio : 1.00571

number of elements : 3163, expected : 1582, actual : 1591, ratio : 1.00569

number of elements : 3173, expected : 1587, actual : 1592, ratio : 1.00315

number of elements : 3183, expected : 1592, actual : 1595, ratio : 1.00188

number of elements : 3193, expected : 1597, actual : 1596, ratio : 0.999374

number of elements : 3203, expected : 1602, actual : 1597, ratio : 0.996879

number of elements : 3213, expected : 1607, actual : 1602, ratio : 0.996889

number of elements : 3223, expected : 1612, actual : 1630, ratio : 1.01117

number of elements : 3233, expected : 1617, actual : 1595, ratio : 0.986395

number of elements : 3243, expected : 1622, actual : 1606, ratio : 0.990136

number of elements : 3253, expected : 1627, actual : 1646, ratio : 1.01168

number of elements : 3263, expected : 1632, actual : 1623, ratio : 0.994485

number of elements : 3273, expected : 1637, actual : 1655, ratio : 1.011

number of elements : 3283, expected : 1642, actual : 1638, ratio : 0.997564

number of elements : 3293, expected : 1647, actual : 1646, ratio : 0.999393

number of elements : 3303, expected : 1652, actual : 1656, ratio : 1.00242

number of elements : 3313, expected : 1657, actual : 1662, ratio : 1.00302

number of elements : 3323, expected : 1662, actual : 1666, ratio : 1.00241

number of elements : 3333, expected : 1667, actual : 1663, ratio : 0.9976

number of elements : 3343, expected : 1672, actual : 1661, ratio : 0.993421

number of elements : 3353, expected : 1677, actual : 1671, ratio : 0.996422

number of elements : 3363, expected : 1682, actual : 1661, ratio : 0.987515

number of elements : 3373, expected : 1687, actual : 1671, ratio : 0.990516

number of elements : 3383, expected : 1692, actual : 1664, ratio : 0.983452

number of elements : 3393, expected : 1697, actual : 1734, ratio : 1.0218

number of elements : 3403, expected : 1702, actual : 1717, ratio : 1.00881

number of elements : 3413, expected : 1707, actual : 1724, ratio : 1.00996

number of elements : 3423, expected : 1712, actual : 1714, ratio : 1.00117

number of elements : 3433, expected : 1717, actual : 1731, ratio : 1.00815

number of elements : 3443, expected : 1722, actual : 1709, ratio : 0.992451

number of elements : 3453, expected : 1727, actual : 1705, ratio : 0.987261

number of elements : 3463, expected : 1732, actual : 1728, ratio : 0.997691

number of elements : 3473, expected : 1737, actual : 1736, ratio : 0.999424

number of elements : 3483, expected : 1742, actual : 1717, ratio : 0.985649

number of elements : 3493, expected : 1747, actual : 1729, ratio : 0.989697

number of elements : 3503, expected : 1752, actual : 1732, ratio : 0.988584

number of elements : 3513, expected : 1757, actual : 1755, ratio : 0.998862

number of elements : 3523, expected : 1762, actual : 1753, ratio : 0.994892

number of elements : 3533, expected : 1767, actual : 1756, ratio : 0.993775

number of elements : 3543, expected : 1772, actual : 1777, ratio : 1.00282

number of elements : 3553, expected : 1777, actual : 1774, ratio : 0.998312

number of elements : 3563, expected : 1782, actual : 1731, ratio : 0.97138

number of elements : 3573, expected : 1787, actual : 1823, ratio : 1.02015

number of elements : 3583, expected : 1792, actual : 1762, ratio : 0.983259

number of elements : 3593, expected : 1797, actual : 1812, ratio : 1.00835

number of elements : 3603, expected : 1802, actual : 1799, ratio : 0.998335

number of elements : 3613, expected : 1807, actual : 1824, ratio : 1.00941

number of elements : 3623, expected : 1812, actual : 1823, ratio : 1.00607

number of elements : 3633, expected : 1817, actual : 1831, ratio : 1.00771

number of elements : 3643, expected : 1822, actual : 1835, ratio : 1.00714

number of elements : 3653, expected : 1827, actual : 1799, ratio : 0.984674

number of elements : 3663, expected : 1832, actual : 1799, ratio : 0.981987

number of elements : 3673, expected : 1837, actual : 1825, ratio : 0.993468

number of elements : 3683, expected : 1842, actual : 1878, ratio : 1.01954

number of elements : 3693, expected : 1847, actual : 1843, ratio : 0.997834

number of elements : 3703, expected : 1852, actual : 1843, ratio : 0.99514

number of elements : 3713, expected : 1857, actual : 1846, ratio : 0.994076

number of elements : 3723, expected : 1862, actual : 1873, ratio : 1.00591

number of elements : 3733, expected : 1867, actual : 1859, ratio : 0.995715

number of elements : 3743, expected : 1872, actual : 1876, ratio : 1.00214

number of elements : 3753, expected : 1877, actual : 1880, ratio : 1.0016

number of elements : 3763, expected : 1882, actual : 1882, ratio : 1

number of elements : 3773, expected : 1887, actual : 1891, ratio : 1.00212

number of elements : 3783, expected : 1892, actual : 1912, ratio : 1.01057

number of elements : 3793, expected : 1897, actual : 1903, ratio : 1.00316

number of elements : 3803, expected : 1902, actual : 1886, ratio : 0.991588

number of elements : 3813, expected : 1907, actual : 1930, ratio : 1.01206

number of elements : 3823, expected : 1912, actual : 1884, ratio : 0.985356

number of elements : 3833, expected : 1917, actual : 1910, ratio : 0.996348

number of elements : 3843, expected : 1922, actual : 1910, ratio : 0.993757

number of elements : 3853, expected : 1927, actual : 1917, ratio : 0.994811

number of elements : 3863, expected : 1932, actual : 1947, ratio : 1.00776

number of elements : 3873, expected : 1937, actual : 1913, ratio : 0.98761

number of elements : 3883, expected : 1942, actual : 1924, ratio : 0.990731

number of elements : 3893, expected : 1947, actual : 1943, ratio : 0.997946

number of elements : 3903, expected : 1952, actual : 1935, ratio : 0.991291

number of elements : 3913, expected : 1957, actual : 1953, ratio : 0.997956

number of elements : 3923, expected : 1962, actual : 1948, ratio : 0.992864

number of elements : 3933, expected : 1967, actual : 1949, ratio : 0.990849

number of elements : 3943, expected : 1972, actual : 1984, ratio : 1.00609

number of elements : 3953, expected : 1977, actual : 1979, ratio : 1.00101

number of elements : 3963, expected : 1982, actual : 2000, ratio : 1.00908

number of elements : 3973, expected : 1987, actual : 2007, ratio : 1.01007

number of elements : 3983, expected : 1992, actual : 1970, ratio : 0.988956

number of elements : 3993, expected : 1997, actual : 2024, ratio : 1.01352

number of elements : 4003, expected : 2002, actual : 1998, ratio : 0.998002

number of elements : 4013, expected : 2007, actual : 1973, ratio : 0.983059

number of elements : 4023, expected : 2012, actual : 2005, ratio : 0.996521

number of elements : 4033, expected : 2017, actual : 2024, ratio : 1.00347

number of elements : 4043, expected : 2022, actual : 2014, ratio : 0.996044

number of elements : 4053, expected : 2027, actual : 2054, ratio : 1.01332

number of elements : 4063, expected : 2032, actual : 2019, ratio : 0.993602

number of elements : 4073, expected : 2037, actual : 1997, ratio : 0.980363

number of elements : 4083, expected : 2042, actual : 2018, ratio : 0.988247

number of elements : 4093, expected : 2047, actual : 2046, ratio : 0.999511

number of elements : 4103, expected : 2052, actual : 2085, ratio : 1.01608

number of elements : 4113, expected : 2057, actual : 2076, ratio : 1.00924

number of elements : 4123, expected : 2062, actual : 2086, ratio : 1.01164

number of elements : 4133, expected : 2067, actual : 2043, ratio : 0.988389

number of elements : 4143, expected : 2072, actual : 2107, ratio : 1.01689

number of elements : 4153, expected : 2077, actual : 2073, ratio : 0.998074

number of elements : 4163, expected : 2082, actual : 2079, ratio : 0.998559

number of elements : 4173, expected : 2087, actual : 2082, ratio : 0.997604

number of elements : 4183, expected : 2092, actual : 2101, ratio : 1.0043

number of elements : 4193, expected : 2097, actual : 2102, ratio : 1.00238

number of elements : 4203, expected : 2102, actual : 2088, ratio : 0.99334

number of elements : 4213, expected : 2107, actual : 2161, ratio : 1.02563

number of elements : 4223, expected : 2112, actual : 2097, ratio : 0.992898

number of elements : 4233, expected : 2117, actual : 2131, ratio : 1.00661

number of elements : 4243, expected : 2122, actual : 2133, ratio : 1.00518

number of elements : 4253, expected : 2127, actual : 2143, ratio : 1.00752

number of elements : 4263, expected : 2132, actual : 2159, ratio : 1.01266

number of elements : 4273, expected : 2137, actual : 2095, ratio : 0.980346

number of elements : 4283, expected : 2142, actual : 2143, ratio : 1.00047

number of elements : 4293, expected : 2147, actual : 2102, ratio : 0.979041

number of elements : 4303, expected : 2152, actual : 2151, ratio : 0.999535

number of elements : 4313, expected : 2157, actual : 2160, ratio : 1.00139

number of elements : 4323, expected : 2162, actual : 2213, ratio : 1.02359

number of elements : 4333, expected : 2167, actual : 2168, ratio : 1.00046

number of elements : 4343, expected : 2172, actual : 2205, ratio : 1.01519

number of elements : 4353, expected : 2177, actual : 2169, ratio : 0.996325

number of elements : 4363, expected : 2182, actual : 2179, ratio : 0.998625

number of elements : 4373, expected : 2187, actual : 2169, ratio : 0.99177

number of elements : 4383, expected : 2192, actual : 2194, ratio : 1.00091

number of elements : 4393, expected : 2197, actual : 2187, ratio : 0.995448

number of elements : 4403, expected : 2202, actual : 2220, ratio : 1.00817

number of elements : 4413, expected : 2207, actual : 2214, ratio : 1.00317

number of elements : 4423, expected : 2212, actual : 2207, ratio : 0.99774

number of elements : 4433, expected : 2217, actual : 2223, ratio : 1.00271

number of elements : 4443, expected : 2222, actual : 2243, ratio : 1.00945

number of elements : 4453, expected : 2227, actual : 2191, ratio : 0.983835

number of elements : 4463, expected : 2232, actual : 2273, ratio : 1.01837

number of elements : 4473, expected : 2237, actual : 2228, ratio : 0.995977

number of elements : 4483, expected : 2242, actual : 2256, ratio : 1.00624

number of elements : 4493, expected : 2247, actual : 2277, ratio : 1.01335

number of elements : 4503, expected : 2252, actual : 2227, ratio : 0.988899

number of elements : 4513, expected : 2257, actual : 2286, ratio : 1.01285

number of elements : 4523, expected : 2262, actual : 2289, ratio : 1.01194

number of elements : 4533, expected : 2267, actual : 2239, ratio : 0.987649

number of elements : 4543, expected : 2272, actual : 2285, ratio : 1.00572

number of elements : 4553, expected : 2277, actual : 2263, ratio : 0.993852

number of elements : 4563, expected : 2282, actual : 2266, ratio : 0.992989

number of elements : 4573, expected : 2287, actual : 2281, ratio : 0.997376

number of elements : 4583, expected : 2292, actual : 2295, ratio : 1.00131

number of elements : 4593, expected : 2297, actual : 2295, ratio : 0.999129

number of elements : 4603, expected : 2302, actual : 2272, ratio : 0.986968

number of elements : 4613, expected : 2307, actual : 2315, ratio : 1.00347

number of elements : 4623, expected : 2312, actual : 2330, ratio : 1.00779

number of elements : 4633, expected : 2317, actual : 2317, ratio : 1

number of elements : 4643, expected : 2322, actual : 2351, ratio : 1.01249

number of elements : 4653, expected : 2327, actual : 2339, ratio : 1.00516

number of elements : 4663, expected : 2332, actual : 2332, ratio : 1

number of elements : 4673, expected : 2337, actual : 2361, ratio : 1.01027

number of elements : 4683, expected : 2342, actual : 2346, ratio : 1.00171

number of elements : 4693, expected : 2347, actual : 2333, ratio : 0.994035

number of elements : 4703, expected : 2352, actual : 2362, ratio : 1.00425

number of elements : 4713, expected : 2357, actual : 2356, ratio : 0.999576

number of elements : 4723, expected : 2362, actual : 2365, ratio : 1.00127

number of elements : 4733, expected : 2367, actual : 2399, ratio : 1.01352

number of elements : 4743, expected : 2372, actual : 2382, ratio : 1.00422

number of elements : 4753, expected : 2377, actual : 2383, ratio : 1.00252

number of elements : 4763, expected : 2382, actual : 2356, ratio : 0.989085

number of elements : 4773, expected : 2387, actual : 2380, ratio : 0.997067

number of elements : 4783, expected : 2392, actual : 2384, ratio : 0.996656

number of elements : 4793, expected : 2397, actual : 2382, ratio : 0.993742

number of elements : 4803, expected : 2402, actual : 2444, ratio : 1.01749

number of elements : 4813, expected : 2407, actual : 2425, ratio : 1.00748

number of elements : 4823, expected : 2412, actual : 2436, ratio : 1.00995

number of elements : 4833, expected : 2417, actual : 2405, ratio : 0.995035

number of elements : 4843, expected : 2422, actual : 2428, ratio : 1.00248

number of elements : 4853, expected : 2427, actual : 2409, ratio : 0.992583

number of elements : 4863, expected : 2432, actual : 2446, ratio : 1.00576

number of elements : 4873, expected : 2437, actual : 2461, ratio : 1.00985

number of elements : 4883, expected : 2442, actual : 2455, ratio : 1.00532

number of elements : 4893, expected : 2447, actual : 2455, ratio : 1.00327

number of elements : 4903, expected : 2452, actual : 2468, ratio : 1.00653

number of elements : 4913, expected : 2457, actual : 2432, ratio : 0.989825

number of elements : 4923, expected : 2462, actual : 2458, ratio : 0.998375

number of elements : 4933, expected : 2467, actual : 2426, ratio : 0.983381

number of elements : 4943, expected : 2472, actual : 2509, ratio : 1.01497

number of elements : 4953, expected : 2477, actual : 2501, ratio : 1.00969

number of elements : 4963, expected : 2482, actual : 2497, ratio : 1.00604

number of elements : 4973, expected : 2487, actual : 2510, ratio : 1.00925

number of elements : 4983, expected : 2492, actual : 2521, ratio : 1.01164

number of elements : 4993, expected : 2497, actual : 2496, ratio : 0.9996

number of elements : 5003, expected : 2502, actual : 2516, ratio : 1.0056

number of elements : 5013, expected : 2507, actual : 2516, ratio : 1.00359

number of elements : 5023, expected : 2512, actual : 2470, ratio : 0.98328

number of elements : 5033, expected : 2517, actual : 2532, ratio : 1.00596

number of elements : 5043, expected : 2522, actual : 2517, ratio : 0.998017

number of elements : 5053, expected : 2527, actual : 2526, ratio : 0.999604

number of elements : 5063, expected : 2532, actual : 2506, ratio : 0.989731

number of elements : 5073, expected : 2537, actual : 2527, ratio : 0.996058

number of elements : 5083, expected : 2542, actual : 2532, ratio : 0.996066

number of elements : 5093, expected : 2547, actual : 2498, ratio : 0.980762

number of elements : 5103, expected : 2552, actual : 2576, ratio : 1.0094

number of elements : 5113, expected : 2557, actual : 2568, ratio : 1.0043

number of elements : 5123, expected : 2562, actual : 2561, ratio : 0.99961

number of elements : 5133, expected : 2567, actual : 2548, ratio : 0.992598

number of elements : 5143, expected : 2572, actual : 2572, ratio : 1

number of elements : 5153, expected : 2577, actual : 2622, ratio : 1.01746

number of elements : 5163, expected : 2582, actual : 2590, ratio : 1.0031

number of elements : 5173, expected : 2587, actual : 2594, ratio : 1.00271

number of elements : 5183, expected : 2592, actual : 2604, ratio : 1.00463

number of elements : 5193, expected : 2597, actual : 2598, ratio : 1.00039

number of elements : 5203, expected : 2602, actual : 2598, ratio : 0.998463

number of elements : 5213, expected : 2607, actual : 2616, ratio : 1.00345

number of elements : 5223, expected : 2612, actual : 2595, ratio : 0.993492

number of elements : 5233, expected : 2617, actual : 2643, ratio : 1.00994

number of elements : 5243, expected : 2622, actual : 2603, ratio : 0.992754

number of elements : 5253, expected : 2627, actual : 2621, ratio : 0.997716

number of elements : 5263, expected : 2632, actual : 2608, ratio : 0.990881

number of elements : 5273, expected : 2637, actual : 2643, ratio : 1.00228

number of elements : 5283, expected : 2642, actual : 2612, ratio : 0.988645

number of elements : 5293, expected : 2647, actual : 2653, ratio : 1.00227

number of elements : 5303, expected : 2652, actual : 2648, ratio : 0.998492

number of elements : 5313, expected : 2657, actual : 2620, ratio : 0.986075

number of elements : 5323, expected : 2662, actual : 2671, ratio : 1.00338

number of elements : 5333, expected : 2667, actual : 2642, ratio : 0.990626

number of elements : 5343, expected : 2672, actual : 2670, ratio : 0.999251

number of elements : 5353, expected : 2677, actual : 2711, ratio : 1.0127

number of elements : 5363, expected : 2682, actual : 2686, ratio : 1.00149

number of elements : 5373, expected : 2687, actual : 2741, ratio : 1.0201

number of elements : 5383, expected : 2692, actual : 2696, ratio : 1.00149

number of elements : 5393, expected : 2697, actual : 2673, ratio : 0.991101

number of elements : 5403, expected : 2702, actual : 2724, ratio : 1.00814

number of elements : 5413, expected : 2707, actual : 2707, ratio : 1

number of elements : 5423, expected : 2712, actual : 2734, ratio : 1.00811

number of elements : 5433, expected : 2717, actual : 2700, ratio : 0.993743

number of elements : 5443, expected : 2722, actual : 2728, ratio : 1.0022

number of elements : 5453, expected : 2727, actual : 2708, ratio : 0.993033

number of elements : 5463, expected : 2732, actual : 2759, ratio : 1.00988

number of elements : 5473, expected : 2737, actual : 2725, ratio : 0.995616

number of elements : 5483, expected : 2742, actual : 2770, ratio : 1.01021

number of elements : 5493, expected : 2747, actual : 2753, ratio : 1.00218

number of elements : 5503, expected : 2752, actual : 2786, ratio : 1.01235

number of elements : 5513, expected : 2757, actual : 2746, ratio : 0.99601

number of elements : 5523, expected : 2762, actual : 2785, ratio : 1.00833

number of elements : 5533, expected : 2767, actual : 2781, ratio : 1.00506

number of elements : 5543, expected : 2772, actual : 2754, ratio : 0.993506

number of elements : 5553, expected : 2777, actual : 2753, ratio : 0.991358

number of elements : 5563, expected : 2782, actual : 2793, ratio : 1.00395

number of elements : 5573, expected : 2787, actual : 2772, ratio : 0.994618

number of elements : 5583, expected : 2792, actual : 2774, ratio : 0.993553

number of elements : 5593, expected : 2797, actual : 2794, ratio : 0.998927

number of elements : 5603, expected : 2802, actual : 2796, ratio : 0.997859

number of elements : 5613, expected : 2807, actual : 2805, ratio : 0.999287

number of elements : 5623, expected : 2812, actual : 2833, ratio : 1.00747

number of elements : 5633, expected : 2817, actual : 2840, ratio : 1.00816

number of elements : 5643, expected : 2822, actual : 2783, ratio : 0.98618

number of elements : 5653, expected : 2827, actual : 2818, ratio : 0.996816

number of elements : 5663, expected : 2832, actual : 2864, ratio : 1.0113

number of elements : 5673, expected : 2837, actual : 2833, ratio : 0.99859

number of elements : 5683, expected : 2842, actual : 2870, ratio : 1.00985

number of elements : 5693, expected : 2847, actual : 2862, ratio : 1.00527

number of elements : 5703, expected : 2852, actual : 2886, ratio : 1.01192

number of elements : 5713, expected : 2857, actual : 2853, ratio : 0.9986

number of elements : 5723, expected : 2862, actual : 2832, ratio : 0.989518

number of elements : 5733, expected : 2867, actual : 2844, ratio : 0.991978

number of elements : 5743, expected : 2872, actual : 2880, ratio : 1.00279

number of elements : 5753, expected : 2877, actual : 2843, ratio : 0.988182

number of elements : 5763, expected : 2882, actual : 2906, ratio : 1.00833

number of elements : 5773, expected : 2887, actual : 2882, ratio : 0.998268

number of elements : 5783, expected : 2892, actual : 2908, ratio : 1.00553

number of elements : 5793, expected : 2897, actual : 2927, ratio : 1.01036

number of elements : 5803, expected : 2902, actual : 2892, ratio : 0.996554

number of elements : 5813, expected : 2907, actual : 2947, ratio : 1.01376

number of elements : 5823, expected : 2912, actual : 2927, ratio : 1.00515

number of elements : 5833, expected : 2917, actual : 2908, ratio : 0.996915

number of elements : 5843, expected : 2922, actual : 2933, ratio : 1.00376

number of elements : 5853, expected : 2927, actual : 2918, ratio : 0.996925

number of elements : 5863, expected : 2932, actual : 2980, ratio : 1.01637

number of elements : 5873, expected : 2937, actual : 2894, ratio : 0.985359

number of elements : 5883, expected : 2942, actual : 2900, ratio : 0.985724

number of elements : 5893, expected : 2947, actual : 2909, ratio : 0.987106

number of elements : 5903, expected : 2952, actual : 2927, ratio : 0.991531

number of elements : 5913, expected : 2957, actual : 2928, ratio : 0.990193

number of elements : 5923, expected : 2962, actual : 2932, ratio : 0.989872

number of elements : 5933, expected : 2967, actual : 2950, ratio : 0.99427

number of elements : 5943, expected : 2972, actual : 2950, ratio : 0.992598

number of elements : 5953, expected : 2977, actual : 2967, ratio : 0.996641

number of elements : 5963, expected : 2982, actual : 3023, ratio : 1.01375

number of elements : 5973, expected : 2987, actual : 2998, ratio : 1.00368

number of elements : 5983, expected : 2992, actual : 2984, ratio : 0.997326

number of elements : 5993, expected : 2997, actual : 2989, ratio : 0.997331

number of elements : 6003, expected : 3002, actual : 3023, ratio : 1.007

number of elements : 6013, expected : 3007, actual : 2986, ratio : 0.993016

number of elements : 6023, expected : 3012, actual : 3056, ratio : 1.01461

number of elements : 6033, expected : 3017, actual : 3025, ratio : 1.00265

number of elements : 6043, expected : 3022, actual : 3019, ratio : 0.999007

number of elements : 6053, expected : 3027, actual : 3021, ratio : 0.998018

number of elements : 6063, expected : 3032, actual : 3001, ratio : 0.989776

number of elements : 6073, expected : 3037, actual : 3048, ratio : 1.00362

number of elements : 6083, expected : 3042, actual : 3054, ratio : 1.00394

number of elements : 6093, expected : 3047, actual : 3062, ratio : 1.00492

number of elements : 6103, expected : 3052, actual : 3078, ratio : 1.00852

number of elements : 6113, expected : 3057, actual : 3058, ratio : 1.00033

number of elements : 6123, expected : 3062, actual : 3058, ratio : 0.998694

number of elements : 6133, expected : 3067, actual : 3092, ratio : 1.00815

number of elements : 6143, expected : 3072, actual : 3045, ratio : 0.991211

number of elements : 6153, expected : 3077, actual : 3072, ratio : 0.998375

number of elements : 6163, expected : 3082, actual : 3045, ratio : 0.987995

number of elements : 6173, expected : 3087, actual : 3098, ratio : 1.00356

number of elements : 6183, expected : 3092, actual : 3144, ratio : 1.01682

number of elements : 6193, expected : 3097, actual : 3094, ratio : 0.999031

number of elements : 6203, expected : 3102, actual : 3117, ratio : 1.00484

number of elements : 6213, expected : 3107, actual : 3108, ratio : 1.00032

number of elements : 6223, expected : 3112, actual : 3107, ratio : 0.998393

number of elements : 6233, expected : 3117, actual : 3116, ratio : 0.999679

number of elements : 6243, expected : 3122, actual : 3113, ratio : 0.997117

number of elements : 6253, expected : 3127, actual : 3109, ratio : 0.994244

number of elements : 6263, expected : 3132, actual : 3169, ratio : 1.01181

number of elements : 6273, expected : 3137, actual : 3123, ratio : 0.995537

number of elements : 6283, expected : 3142, actual : 3117, ratio : 0.992043

number of elements : 6293, expected : 3147, actual : 3120, ratio : 0.99142

number of elements : 6303, expected : 3152, actual : 3154, ratio : 1.00063

number of elements : 6313, expected : 3157, actual : 3143, ratio : 0.995565

number of elements : 6323, expected : 3162, actual : 3157, ratio : 0.998419

number of elements : 6333, expected : 3167, actual : 3179, ratio : 1.00379

number of elements : 6343, expected : 3172, actual : 3168, ratio : 0.998739

number of elements : 6353, expected : 3177, actual : 3166, ratio : 0.996538

number of elements : 6363, expected : 3182, actual : 3160, ratio : 0.993086

number of elements : 6373, expected : 3187, actual : 3195, ratio : 1.00251

number of elements : 6383, expected : 3192, actual : 3185, ratio : 0.997807

number of elements : 6393, expected : 3197, actual : 3197, ratio : 1

number of elements : 6403, expected : 3202, actual : 3213, ratio : 1.00344

number of elements : 6413, expected : 3207, actual : 3204, ratio : 0.999065

number of elements : 6423, expected : 3212, actual : 3182, ratio : 0.99066

number of elements : 6433, expected : 3217, actual : 3182, ratio : 0.98912

number of elements : 6443, expected : 3222, actual : 3243, ratio : 1.00652

number of elements : 6453, expected : 3227, actual : 3239, ratio : 1.00372

number of elements : 6463, expected : 3232, actual : 3255, ratio : 1.00712

number of elements : 6473, expected : 3237, actual : 3288, ratio : 1.01576

number of elements : 6483, expected : 3242, actual : 3259, ratio : 1.00524

number of elements : 6493, expected : 3247, actual : 3227, ratio : 0.99384

number of elements : 6503, expected : 3252, actual : 3303, ratio : 1.01568

number of elements : 6513, expected : 3257, actual : 3260, ratio : 1.00092

number of elements : 6523, expected : 3262, actual : 3284, ratio : 1.00674

number of elements : 6533, expected : 3267, actual : 3281, ratio : 1.00429

number of elements : 6543, expected : 3272, actual : 3296, ratio : 1.00733

number of elements : 6553, expected : 3277, actual : 3245, ratio : 0.990235

number of elements : 6563, expected : 3282, actual : 3240, ratio : 0.987203

number of elements : 6573, expected : 3287, actual : 3307, ratio : 1.00608

number of elements : 6583, expected : 3292, actual : 3287, ratio : 0.998481

number of elements : 6593, expected : 3297, actual : 3268, ratio : 0.991204

number of elements : 6603, expected : 3302, actual : 3309, ratio : 1.00212

number of elements : 6613, expected : 3307, actual : 3296, ratio : 0.996674

number of elements : 6623, expected : 3312, actual : 3336, ratio : 1.00725

number of elements : 6633, expected : 3317, actual : 3307, ratio : 0.996985

number of elements : 6643, expected : 3322, actual : 3321, ratio : 0.999699

number of elements : 6653, expected : 3327, actual : 3280, ratio : 0.985873

number of elements : 6663, expected : 3332, actual : 3370, ratio : 1.0114

number of elements : 6673, expected : 3337, actual : 3367, ratio : 1.00899

number of elements : 6683, expected : 3342, actual : 3357, ratio : 1.00449

number of elements : 6693, expected : 3347, actual : 3356, ratio : 1.00269

number of elements : 6703, expected : 3352, actual : 3367, ratio : 1.00447

number of elements : 6713, expected : 3357, actual : 3392, ratio : 1.01043

number of elements : 6723, expected : 3362, actual : 3348, ratio : 0.995836

number of elements : 6733, expected : 3367, actual : 3385, ratio : 1.00535

number of elements : 6743, expected : 3372, actual : 3380, ratio : 1.00237

number of elements : 6753, expected : 3377, actual : 3353, ratio : 0.992893

number of elements : 6763, expected : 3382, actual : 3356, ratio : 0.992312

number of elements : 6773, expected : 3387, actual : 3377, ratio : 0.997048

number of elements : 6783, expected : 3392, actual : 3400, ratio : 1.00236

number of elements : 6793, expected : 3397, actual : 3416, ratio : 1.00559

number of elements : 6803, expected : 3402, actual : 3407, ratio : 1.00147

number of elements : 6813, expected : 3407, actual : 3413, ratio : 1.00176

number of elements : 6823, expected : 3412, actual : 3401, ratio : 0.996776

number of elements : 6833, expected : 3417, actual : 3405, ratio : 0.996488

number of elements : 6843, expected : 3422, actual : 3431, ratio : 1.00263

number of elements : 6853, expected : 3427, actual : 3445, ratio : 1.00525

number of elements : 6863, expected : 3432, actual : 3421, ratio : 0.996795

number of elements : 6873, expected : 3437, actual : 3446, ratio : 1.00262

number of elements : 6883, expected : 3442, actual : 3413, ratio : 0.991575

number of elements : 6893, expected : 3447, actual : 3432, ratio : 0.995648

number of elements : 6903, expected : 3452, actual : 3439, ratio : 0.996234

number of elements : 6913, expected : 3457, actual : 3471, ratio : 1.00405

number of elements : 6923, expected : 3462, actual : 3452, ratio : 0.997111

number of elements : 6933, expected : 3467, actual : 3451, ratio : 0.995385

number of elements : 6943, expected : 3472, actual : 3488, ratio : 1.00461

number of elements : 6953, expected : 3477, actual : 3469, ratio : 0.997699

number of elements : 6963, expected : 3482, actual : 3453, ratio : 0.991671

number of elements : 6973, expected : 3487, actual : 3497, ratio : 1.00287

number of elements : 6983, expected : 3492, actual : 3475, ratio : 0.995132

number of elements : 6993, expected : 3497, actual : 3466, ratio : 0.991135

number of elements : 7003, expected : 3502, actual : 3488, ratio : 0.996002

number of elements : 7013, expected : 3507, actual : 3503, ratio : 0.998859

number of elements : 7023, expected : 3512, actual : 3497, ratio : 0.995729

number of elements : 7033, expected : 3517, actual : 3533, ratio : 1.00455

number of elements : 7043, expected : 3522, actual : 3505, ratio : 0.995173

number of elements : 7053, expected : 3527, actual : 3513, ratio : 0.996031

number of elements : 7063, expected : 3532, actual : 3552, ratio : 1.00566

number of elements : 7073, expected : 3537, actual : 3592, ratio : 1.01555

number of elements : 7083, expected : 3542, actual : 3537, ratio : 0.998588

number of elements : 7093, expected : 3547, actual : 3514, ratio : 0.990696

number of elements : 7103, expected : 3552, actual : 3531, ratio : 0.994088

number of elements : 7113, expected : 3557, actual : 3594, ratio : 1.0104

number of elements : 7123, expected : 3562, actual : 3566, ratio : 1.00112

number of elements : 7133, expected : 3567, actual : 3576, ratio : 1.00252

number of elements : 7143, expected : 3572, actual : 3609, ratio : 1.01036

number of elements : 7153, expected : 3577, actual : 3595, ratio : 1.00503

number of elements : 7163, expected : 3582, actual : 3605, ratio : 1.00642

number of elements : 7173, expected : 3587, actual : 3572, ratio : 0.995818

number of elements : 7183, expected : 3592, actual : 3631, ratio : 1.01086

number of elements : 7193, expected : 3597, actual : 3606, ratio : 1.0025

number of elements : 7203, expected : 3602, actual : 3550, ratio : 0.985564

number of elements : 7213, expected : 3607, actual : 3607, ratio : 1

number of elements : 7223, expected : 3612, actual : 3619, ratio : 1.00194

number of elements : 7233, expected : 3617, actual : 3637, ratio : 1.00553

number of elements : 7243, expected : 3622, actual : 3641, ratio : 1.00525

number of elements : 7253, expected : 3627, actual : 3578, ratio : 0.98649

number of elements : 7263, expected : 3632, actual : 3616, ratio : 0.995595

number of elements : 7273, expected : 3637, actual : 3627, ratio : 0.99725

number of elements : 7283, expected : 3642, actual : 3630, ratio : 0.996705

number of elements : 7293, expected : 3647, actual : 3610, ratio : 0.989855

number of elements : 7303, expected : 3652, actual : 3627, ratio : 0.993154

number of elements : 7313, expected : 3657, actual : 3669, ratio : 1.00328

number of elements : 7323, expected : 3662, actual : 3648, ratio : 0.996177

number of elements : 7333, expected : 3667, actual : 3675, ratio : 1.00218

number of elements : 7343, expected : 3672, actual : 3687, ratio : 1.00408

number of elements : 7353, expected : 3677, actual : 3709, ratio : 1.0087

number of elements : 7363, expected : 3682, actual : 3701, ratio : 1.00516

number of elements : 7373, expected : 3687, actual : 3682, ratio : 0.998644

number of elements : 7383, expected : 3692, actual : 3685, ratio : 0.998104

number of elements : 7393, expected : 3697, actual : 3710, ratio : 1.00352

number of elements : 7403, expected : 3702, actual : 3731, ratio : 1.00783

number of elements : 7413, expected : 3707, actual : 3663, ratio : 0.988131

number of elements : 7423, expected : 3712, actual : 3758, ratio : 1.01239

number of elements : 7433, expected : 3717, actual : 3754, ratio : 1.00995

number of elements : 7443, expected : 3722, actual : 3726, ratio : 1.00107

number of elements : 7453, expected : 3727, actual : 3698, ratio : 0.992219

number of elements : 7463, expected : 3732, actual : 3731, ratio : 0.999732

number of elements : 7473, expected : 3737, actual : 3685, ratio : 0.986085

number of elements : 7483, expected : 3742, actual : 3709, ratio : 0.991181

number of elements : 7493, expected : 3747, actual : 3759, ratio : 1.0032

number of elements : 7503, expected : 3752, actual : 3736, ratio : 0.995736

number of elements : 7513, expected : 3757, actual : 3729, ratio : 0.992547

number of elements : 7523, expected : 3762, actual : 3832, ratio : 1.01861

number of elements : 7533, expected : 3767, actual : 3777, ratio : 1.00265

number of elements : 7543, expected : 3772, actual : 3752, ratio : 0.994698

number of elements : 7553, expected : 3777, actual : 3806, ratio : 1.00768

number of elements : 7563, expected : 3782, actual : 3776, ratio : 0.998414

number of elements : 7573, expected : 3787, actual : 3805, ratio : 1.00475

number of elements : 7583, expected : 3792, actual : 3784, ratio : 0.99789

number of elements : 7593, expected : 3797, actual : 3813, ratio : 1.00421

number of elements : 7603, expected : 3802, actual : 3813, ratio : 1.00289

number of elements : 7613, expected : 3807, actual : 3849, ratio : 1.01103

number of elements : 7623, expected : 3812, actual : 3818, ratio : 1.00157

number of elements : 7633, expected : 3817, actual : 3832, ratio : 1.00393

number of elements : 7643, expected : 3822, actual : 3825, ratio : 1.00078

number of elements : 7653, expected : 3827, actual : 3881, ratio : 1.01411

number of elements : 7663, expected : 3832, actual : 3810, ratio : 0.994259

number of elements : 7673, expected : 3837, actual : 3816, ratio : 0.994527

number of elements : 7683, expected : 3842, actual : 3882, ratio : 1.01041

number of elements : 7693, expected : 3847, actual : 3855, ratio : 1.00208

number of elements : 7703, expected : 3852, actual : 3860, ratio : 1.00208

number of elements : 7713, expected : 3857, actual : 3817, ratio : 0.989629

number of elements : 7723, expected : 3862, actual : 3857, ratio : 0.998705

number of elements : 7733, expected : 3867, actual : 3917, ratio : 1.01293

number of elements : 7743, expected : 3872, actual : 3865, ratio : 0.998192

number of elements : 7753, expected : 3877, actual : 3867, ratio : 0.997421

number of elements : 7763, expected : 3882, actual : 3895, ratio : 1.00335

number of elements : 7773, expected : 3887, actual : 3867, ratio : 0.994855

number of elements : 7783, expected : 3892, actual : 3930, ratio : 1.00976

number of elements : 7793, expected : 3897, actual : 3901, ratio : 1.00103

number of elements : 7803, expected : 3902, actual : 3870, ratio : 0.991799

number of elements : 7813, expected : 3907, actual : 3920, ratio : 1.00333

number of elements : 7823, expected : 3912, actual : 3926, ratio : 1.00358

number of elements : 7833, expected : 3917, actual : 3903, ratio : 0.996426

number of elements : 7843, expected : 3922, actual : 3930, ratio : 1.00204

number of elements : 7853, expected : 3927, actual : 3939, ratio : 1.00306

number of elements : 7863, expected : 3932, actual : 3964, ratio : 1.00814

number of elements : 7873, expected : 3937, actual : 3909, ratio : 0.992888

number of elements : 7883, expected : 3942, actual : 3914, ratio : 0.992897

number of elements : 7893, expected : 3947, actual : 3977, ratio : 1.0076

number of elements : 7903, expected : 3952, actual : 3939, ratio : 0.996711

number of elements : 7913, expected : 3957, actual : 3959, ratio : 1.00051

number of elements : 7923, expected : 3962, actual : 3951, ratio : 0.997224

number of elements : 7933, expected : 3967, actual : 3963, ratio : 0.998992

number of elements : 7943, expected : 3972, actual : 3967, ratio : 0.998741

number of elements : 7953, expected : 3977, actual : 4002, ratio : 1.00629

number of elements : 7963, expected : 3982, actual : 3981, ratio : 0.999749

number of elements : 7973, expected : 3987, actual : 3969, ratio : 0.995485

number of elements : 7983, expected : 3992, actual : 4040, ratio : 1.01202

number of elements : 7993, expected : 3997, actual : 4050, ratio : 1.01326

number of elements : 8003, expected : 4002, actual : 3958, ratio : 0.989005

number of elements : 8013, expected : 4007, actual : 4023, ratio : 1.00399

number of elements : 8023, expected : 4012, actual : 4045, ratio : 1.00823

number of elements : 8033, expected : 4017, actual : 3967, ratio : 0.987553

number of elements : 8043, expected : 4022, actual : 4018, ratio : 0.999005

number of elements : 8053, expected : 4027, actual : 4015, ratio : 0.99702

number of elements : 8063, expected : 4032, actual : 4002, ratio : 0.99256

number of elements : 8073, expected : 4037, actual : 4063, ratio : 1.00644

number of elements : 8083, expected : 4042, actual : 4068, ratio : 1.00643

number of elements : 8093, expected : 4047, actual : 4036, ratio : 0.997282

number of elements : 8103, expected : 4052, actual : 4089, ratio : 1.00913

number of elements : 8113, expected : 4057, actual : 4027, ratio : 0.992605

number of elements : 8123, expected : 4062, actual : 4060, ratio : 0.999508

number of elements : 8133, expected : 4067, actual : 4035, ratio : 0.992132

number of elements : 8143, expected : 4072, actual : 4128, ratio : 1.01375

number of elements : 8153, expected : 4077, actual : 4081, ratio : 1.00098

number of elements : 8163, expected : 4082, actual : 4027, ratio : 0.986526

number of elements : 8173, expected : 4087, actual : 4132, ratio : 1.01101

number of elements : 8183, expected : 4092, actual : 4111, ratio : 1.00464

number of elements : 8193, expected : 4097, actual : 4081, ratio : 0.996095

number of elements : 8203, expected : 4102, actual : 4080, ratio : 0.994637

number of elements : 8213, expected : 4107, actual : 4105, ratio : 0.999513

number of elements : 8223, expected : 4112, actual : 4167, ratio : 1.01338

number of elements : 8233, expected : 4117, actual : 4127, ratio : 1.00243

number of elements : 8243, expected : 4122, actual : 4158, ratio : 1.00873

number of elements : 8253, expected : 4127, actual : 4135, ratio : 1.00194

number of elements : 8263, expected : 4132, actual : 4185, ratio : 1.01283

number of elements : 8273, expected : 4137, actual : 4161, ratio : 1.0058

number of elements : 8283, expected : 4142, actual : 4172, ratio : 1.00724

number of elements : 8293, expected : 4147, actual : 4122, ratio : 0.993972

number of elements : 8303, expected : 4152, actual : 4150, ratio : 0.999518

number of elements : 8313, expected : 4157, actual : 4139, ratio : 0.99567

number of elements : 8323, expected : 4162, actual : 4224, ratio : 1.0149

number of elements : 8333, expected : 4167, actual : 4138, ratio : 0.993041

number of elements : 8343, expected : 4172, actual : 4164, ratio : 0.998082

number of elements : 8353, expected : 4177, actual : 4107, ratio : 0.983242

number of elements : 8363, expected : 4182, actual : 4158, ratio : 0.994261

number of elements : 8373, expected : 4187, actual : 4157, ratio : 0.992835

number of elements : 8383, expected : 4192, actual : 4152, ratio : 0.990458

number of elements : 8393, expected : 4197, actual : 4186, ratio : 0.997379

number of elements : 8403, expected : 4202, actual : 4263, ratio : 1.01452

number of elements : 8413, expected : 4207, actual : 4219, ratio : 1.00285

number of elements : 8423, expected : 4212, actual : 4211, ratio : 0.999763

number of elements : 8433, expected : 4217, actual : 4170, ratio : 0.988855

number of elements : 8443, expected : 4222, actual : 4199, ratio : 0.994552

number of elements : 8453, expected : 4227, actual : 4196, ratio : 0.992666

number of elements : 8463, expected : 4232, actual : 4190, ratio : 0.990076

number of elements : 8473, expected : 4237, actual : 4240, ratio : 1.00071

number of elements : 8483, expected : 4242, actual : 4245, ratio : 1.00071

number of elements : 8493, expected : 4247, actual : 4228, ratio : 0.995526

number of elements : 8503, expected : 4252, actual : 4281, ratio : 1.00682

number of elements : 8513, expected : 4257, actual : 4272, ratio : 1.00352

number of elements : 8523, expected : 4262, actual : 4259, ratio : 0.999296

number of elements : 8533, expected : 4267, actual : 4244, ratio : 0.99461

number of elements : 8543, expected : 4272, actual : 4266, ratio : 0.998596

number of elements : 8553, expected : 4277, actual : 4344, ratio : 1.01567

number of elements : 8563, expected : 4282, actual : 4241, ratio : 0.990425

number of elements : 8573, expected : 4287, actual : 4304, ratio : 1.00397

number of elements : 8583, expected : 4292, actual : 4283, ratio : 0.997903

number of elements : 8593, expected : 4297, actual : 4266, ratio : 0.992786

number of elements : 8603, expected : 4302, actual : 4284, ratio : 0.995816

number of elements : 8613, expected : 4307, actual : 4269, ratio : 0.991177

number of elements : 8623, expected : 4312, actual : 4299, ratio : 0.996985

number of elements : 8633, expected : 4317, actual : 4278, ratio : 0.990966

number of elements : 8643, expected : 4322, actual : 4307, ratio : 0.996529

number of elements : 8653, expected : 4327, actual : 4332, ratio : 1.00116

number of elements : 8663, expected : 4332, actual : 4374, ratio : 1.0097

number of elements : 8673, expected : 4337, actual : 4352, ratio : 1.00346

number of elements : 8683, expected : 4342, actual : 4349, ratio : 1.00161

number of elements : 8693, expected : 4347, actual : 4334, ratio : 0.997009

number of elements : 8703, expected : 4352, actual : 4363, ratio : 1.00253

number of elements : 8713, expected : 4357, actual : 4357, ratio : 1

number of elements : 8723, expected : 4362, actual : 4349, ratio : 0.99702

number of elements : 8733, expected : 4367, actual : 4330, ratio : 0.991527

number of elements : 8743, expected : 4372, actual : 4327, ratio : 0.989707

number of elements : 8753, expected : 4377, actual : 4359, ratio : 0.995888

number of elements : 8763, expected : 4382, actual : 4392, ratio : 1.00228

number of elements : 8773, expected : 4387, actual : 4394, ratio : 1.0016

number of elements : 8783, expected : 4392, actual : 4353, ratio : 0.99112

number of elements : 8793, expected : 4397, actual : 4400, ratio : 1.00068

number of elements : 8803, expected : 4402, actual : 4416, ratio : 1.00318

number of elements : 8813, expected : 4407, actual : 4429, ratio : 1.00499

number of elements : 8823, expected : 4412, actual : 4463, ratio : 1.01156

number of elements : 8833, expected : 4417, actual : 4425, ratio : 1.00181

number of elements : 8843, expected : 4422, actual : 4401, ratio : 0.995251

number of elements : 8853, expected : 4427, actual : 4405, ratio : 0.99503

number of elements : 8863, expected : 4432, actual : 4399, ratio : 0.992554

number of elements : 8873, expected : 4437, actual : 4409, ratio : 0.993689

number of elements : 8883, expected : 4442, actual : 4462, ratio : 1.0045

number of elements : 8893, expected : 4447, actual : 4453, ratio : 1.00135

number of elements : 8903, expected : 4452, actual : 4424, ratio : 0.993711

number of elements : 8913, expected : 4457, actual : 4501, ratio : 1.00987

number of elements : 8923, expected : 4462, actual : 4454, ratio : 0.998207

number of elements : 8933, expected : 4467, actual : 4519, ratio : 1.01164

number of elements : 8943, expected : 4472, actual : 4476, ratio : 1.00089

number of elements : 8953, expected : 4477, actual : 4521, ratio : 1.00983

number of elements : 8963, expected : 4482, actual : 4587, ratio : 1.02343

number of elements : 8973, expected : 4487, actual : 4474, ratio : 0.997103

number of elements : 8983, expected : 4492, actual : 4496, ratio : 1.00089

number of elements : 8993, expected : 4497, actual : 4511, ratio : 1.00311

number of elements : 9003, expected : 4502, actual : 4470, ratio : 0.992892

number of elements : 9013, expected : 4507, actual : 4507, ratio : 1

number of elements : 9023, expected : 4512, actual : 4504, ratio : 0.998227

number of elements : 9033, expected : 4517, actual : 4532, ratio : 1.00332

number of elements : 9043, expected : 4522, actual : 4549, ratio : 1.00597

number of elements : 9053, expected : 4527, actual : 4479, ratio : 0.989397

number of elements : 9063, expected : 4532, actual : 4525, ratio : 0.998455

number of elements : 9073, expected : 4537, actual : 4529, ratio : 0.998237

number of elements : 9083, expected : 4542, actual : 4517, ratio : 0.994496

number of elements : 9093, expected : 4547, actual : 4551, ratio : 1.00088

number of elements : 9103, expected : 4552, actual : 4569, ratio : 1.00373

number of elements : 9113, expected : 4557, actual : 4599, ratio : 1.00922

number of elements : 9123, expected : 4562, actual : 4566, ratio : 1.00088

number of elements : 9133, expected : 4567, actual : 4561, ratio : 0.998686

number of elements : 9143, expected : 4572, actual : 4559, ratio : 0.997157

number of elements : 9153, expected : 4577, actual : 4569, ratio : 0.998252

number of elements : 9163, expected : 4582, actual : 4546, ratio : 0.992143

number of elements : 9173, expected : 4587, actual : 4602, ratio : 1.00327

number of elements : 9183, expected : 4592, actual : 4604, ratio : 1.00261

number of elements : 9193, expected : 4597, actual : 4606, ratio : 1.00196

number of elements : 9203, expected : 4602, actual : 4580, ratio : 0.995219

number of elements : 9213, expected : 4607, actual : 4590, ratio : 0.99631

number of elements : 9223, expected : 4612, actual : 4669, ratio : 1.01236

number of elements : 9233, expected : 4617, actual : 4600, ratio : 0.996318

number of elements : 9243, expected : 4622, actual : 4619, ratio : 0.999351

number of elements : 9253, expected : 4627, actual : 4672, ratio : 1.00973

number of elements : 9263, expected : 4632, actual : 4648, ratio : 1.00345

number of elements : 9273, expected : 4637, actual : 4650, ratio : 1.0028

number of elements : 9283, expected : 4642, actual : 4668, ratio : 1.0056

number of elements : 9293, expected : 4647, actual : 4676, ratio : 1.00624

number of elements : 9303, expected : 4652, actual : 4655, ratio : 1.00064

number of elements : 9313, expected : 4657, actual : 4659, ratio : 1.00043

number of elements : 9323, expected : 4662, actual : 4711, ratio : 1.01051

number of elements : 9333, expected : 4667, actual : 4682, ratio : 1.00321

number of elements : 9343, expected : 4672, actual : 4666, ratio : 0.998716

number of elements : 9353, expected : 4677, actual : 4727, ratio : 1.01069

number of elements : 9363, expected : 4682, actual : 4699, ratio : 1.00363

number of elements : 9373, expected : 4687, actual : 4710, ratio : 1.00491

number of elements : 9383, expected : 4692, actual : 4705, ratio : 1.00277

number of elements : 9393, expected : 4697, actual : 4624, ratio : 0.984458

number of elements : 9403, expected : 4702, actual : 4704, ratio : 1.00043

number of elements : 9413, expected : 4707, actual : 4744, ratio : 1.00786

number of elements : 9423, expected : 4712, actual : 4733, ratio : 1.00446

number of elements : 9433, expected : 4717, actual : 4736, ratio : 1.00403

number of elements : 9443, expected : 4722, actual : 4694, ratio : 0.99407

number of elements : 9453, expected : 4727, actual : 4681, ratio : 0.990269

number of elements : 9463, expected : 4732, actual : 4783, ratio : 1.01078

number of elements : 9473, expected : 4737, actual : 4706, ratio : 0.993456

number of elements : 9483, expected : 4742, actual : 4719, ratio : 0.99515

number of elements : 9493, expected : 4747, actual : 4760, ratio : 1.00274

number of elements : 9503, expected : 4752, actual : 4785, ratio : 1.00694

number of elements : 9513, expected : 4757, actual : 4760, ratio : 1.00063

number of elements : 9523, expected : 4762, actual : 4775, ratio : 1.00273

number of elements : 9533, expected : 4767, actual : 4713, ratio : 0.988672

number of elements : 9543, expected : 4772, actual : 4773, ratio : 1.00021

number of elements : 9553, expected : 4777, actual : 4772, ratio : 0.998953

number of elements : 9563, expected : 4782, actual : 4744, ratio : 0.992054

number of elements : 9573, expected : 4787, actual : 4826, ratio : 1.00815

number of elements : 9583, expected : 4792, actual : 4771, ratio : 0.995618

number of elements : 9593, expected : 4797, actual : 4792, ratio : 0.998958

number of elements : 9603, expected : 4802, actual : 4828, ratio : 1.00541

number of elements : 9613, expected : 4807, actual : 4825, ratio : 1.00374

number of elements : 9623, expected : 4812, actual : 4835, ratio : 1.00478

number of elements : 9633, expected : 4817, actual : 4834, ratio : 1.00353

number of elements : 9643, expected : 4822, actual : 4808, ratio : 0.997097

number of elements : 9653, expected : 4827, actual : 4859, ratio : 1.00663

number of elements : 9663, expected : 4832, actual : 4833, ratio : 1.00021

number of elements : 9673, expected : 4837, actual : 4808, ratio : 0.994005

number of elements : 9683, expected : 4842, actual : 4810, ratio : 0.993391

number of elements : 9693, expected : 4847, actual : 4854, ratio : 1.00144

number of elements : 9703, expected : 4852, actual : 4841, ratio : 0.997733

number of elements : 9713, expected : 4857, actual : 4842, ratio : 0.996912

number of elements : 9723, expected : 4862, actual : 4838, ratio : 0.995064

number of elements : 9733, expected : 4867, actual : 4824, ratio : 0.991165

number of elements : 9743, expected : 4872, actual : 4839, ratio : 0.993227

number of elements : 9753, expected : 4877, actual : 4842, ratio : 0.992823

number of elements : 9763, expected : 4882, actual : 4859, ratio : 0.995289

number of elements : 9773, expected : 4887, actual : 4878, ratio : 0.998158

number of elements : 9783, expected : 4892, actual : 4832, ratio : 0.987735

number of elements : 9793, expected : 4897, actual : 4935, ratio : 1.00776

number of elements : 9803, expected : 4902, actual : 4934, ratio : 1.00653

number of elements : 9813, expected : 4907, actual : 4882, ratio : 0.994905

number of elements : 9823, expected : 4912, actual : 4911, ratio : 0.999796

number of elements : 9833, expected : 4917, actual : 4910, ratio : 0.998576

number of elements : 9843, expected : 4922, actual : 4934, ratio : 1.00244

number of elements : 9853, expected : 4927, actual : 4888, ratio : 0.992084

number of elements : 9863, expected : 4932, actual : 4933, ratio : 1.0002

number of elements : 9873, expected : 4937, actual : 4962, ratio : 1.00506

number of elements : 9883, expected : 4942, actual : 4980, ratio : 1.00769

number of elements : 9893, expected : 4947, actual : 4915, ratio : 0.993531

number of elements : 9903, expected : 4952, actual : 4910, ratio : 0.991519

number of elements : 9913, expected : 4957, actual : 4956, ratio : 0.999798

number of elements : 9923, expected : 4962, actual : 4962, ratio : 1

number of elements : 9933, expected : 4967, actual : 4986, ratio : 1.00383

number of elements : 9943, expected : 4972, actual : 4962, ratio : 0.997989

number of elements : 9953, expected : 4977, actual : 4946, ratio : 0.993771

number of elements : 9963, expected : 4982, actual : 4959, ratio : 0.995383

number of elements : 9973, expected : 4987, actual : 4971, ratio : 0.996792

number of elements : 9983, expected : 4992, actual : 5038, ratio : 1.00921

number of elements : 9993, expected : 4997, actual : 4971, ratio : 0.994797