# Expected Number of ordered sub-arrays

## Definitions

Ordered Sub-array :

Given an array ***A*** of length ***n***, the range ***[i, j)*** is said to be an Ordered Sub-array of ***A***, if

1. ***A[k+1] > A[k]***, for ***i <= k < j***;

2. ***A[i – 1] > A[i]***, if ***i > 0***;

3. ***A[j – 1] > A[j]***, if ***j < n***;

## Problems

An array ***A*** is said to be random if the element in ***A*** is random and the order of the element is random. Assume array ***A*** is random and the elements in it are distinct, what is the expected number of Ordered Sub-arrays in ***A***?

## Solutions

A permutation of an array is an ordering of the elements in the array.

For a random array of length ***n*** with distinct elements, all the ***n!*** permutations of the array are equally likely to occur.

Let denote the number of permutations of an array of length ***n*** , where each of the permutations has ***i*** Ordered Sub-arrays.

Let ***A*** be an array of length ***(n+1)***.

Let ***A'*** be the array formed from ***A*** by removing the smallest element in ***A***.

Let denote a permutation of ***A***, where there are ***i*** Ordered Sub-arrays.

Let denote a permutation of ***A'***, where there are ***i*** Ordered Sub-arrays.

For every . If the smallest element is inserted at the start of an Ordered Sub-array in , we will get , there are ***i*** such cases. If the smallest element is inserted at the other positions instead of the start of an Ordered Sub-array, we will get , there are ***(n + 2 – i)*** such cases.

For every . If the smallest element is removed from , we will either get or .

Since is the number of , is the number of , andis the number of , follow from the above reasoning:

***Equation 1***

Let denote the possibility that there are ***i*** Ordered Sub-arrays in a random array, then , by “***Equation 1***” :

***Equation 2***

Let ***E(n)*** denote the expected number of Ordered Sub-arrays in a random array, then

, by “***Equation 2***”:

That is the expected number of Ordered Sub-arrays in an array of length ***n*** is ***(n+1)/2***.